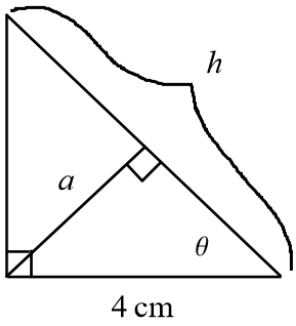
Problem 1

One of the legs of a right triangle has length 4 cm. Express the length of the altitude perpendicular to the hypotenuse as a function of the length of the hypotenuse.

Solution

Draw a schematic of the right triangle with the altitude perpendicular to the hypotenuse.



Take the sine and cosine of the labelled angle θ.

$$\begin{cases} \cos\theta = \dfrac{4}{h} \\ \sin\theta = \dfrac{a}{4} \end{cases}$$

Square both sides of each equation.

$$\begin{cases} \cos^2\theta = \dfrac{16}{h^2} \\ \sin^2\theta = \dfrac{a^2}{16} \end{cases}$$

Add the respective sides of these equations.

$$\cos^2\theta + \sin^2\theta = \frac{16}{h^2} + \frac{a^2}{16}$$

Use the fact that $\cos^2\theta + \sin^2\theta = 1$ to eliminate θ.

$$1 = \frac{16}{h^2} + \frac{a^2}{16}$$

Solve for a^2.

$$a^2 = \frac{16}{h^2}(h^2 - 16)$$

Therefore, taking the square root of both sides, the length of the altitude in centimeters is

$$a = \frac{4}{h}\sqrt{h^2 - 16}.$$